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Connection between the slave-particle and x-operator path-integral representations. A new perturbative approach

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Abstract. In this paper it is shown that the family of first-order Lagrangians for the t–J model and the corresponding correlation-generating functional previously found can be exactly mapped into the slave-fermion decoupled representation. Next, by means of the Faddeev–Jackiw symplectic method, a different family of Lagrangians is constructed and it is shown how the corresponding correlation-generating functional can be mapped into the slave-boson representation. Finally, in order to define the propagation of fermion modes we discuss two alternative ways to treat the fermionic sector in the path-integral formalism for the t–J model.

1. Introduction

Due to its relevance in describing the behaviour of strongly correlated electron systems, there has been renewed interest in the study of supersymmetric generalizations of the Hubbard model over the last few years. A complete review on strongly correlated electron systems together with its connection to high-T_c superconductors is given in [1].

The Hubbard models based on the superalgebras $spl(2, 1)$, $osp(2, 2)$ or $su(2, 2)$ have been formulated using several approaches [2–6]. For instance, as suggested in [7–9], the superalgebra $spl(2, 1)$ could be useful for studying the model in the limit of infinite on-site repulsion and with infinite-range hopping between all sites [10].

Many problems concerning correlated electron systems have been treated within the framework of the decoupled slave-particle representations. Two of them are most important: the slave-boson and the slave-fermion representations. The first one favours the fermion dynamics, and therefore the slave-boson representation seems to be better for describing a Fermi liquid state [11, 12]. Instead, the slave-fermion representation seems to give a good response when the system is closed to the antiferromagnetic order [13, 14].

An important question in order to understand the physics of high-T_c superconductors, is to solve the problem of how to go from one representation to the other. This is because in high-T_c superconductors, both Fermi liquid and magnetic order states seem to be present.

On the other hand, one of the main problems appearing in these kinds of models is to define the dynamics of fermions in the constrained Hilbert space, when double occupancy of lattice sites is excluded. In this case a convenient representation is also given in terms of slave-particles [15].

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As is known the slave-particle models exhibit a local gauge invariance which is destroyed in the mean-field approximation. This local gauge invariance has an associated first-class constraint which is difficult to handle in the path-integral formalism.

Another possible way to attack the problem was given in [16, 17] by using generalized coherent states within the framework of the functional integral formalism.

Recently, the t–J model was analysed in the context of the path-integral formalism [18, 19]. Our starting point was the construction of a particular family of first-order constrained Lagrangians using the Faddeev–Jackiw (FJ) symplectic method [20], in the supersymmetric version [21, 22]. In this approach any decoupling is used, but the field variables are directly the Hubbard X-operators which verify the superalgebra $spl(2, 1)$. In this way, we always work with real physical excitations.

Next, by using path-integral techniques, the correlation generating functional and the effective Lagrangian were constructed. Moreover, we have proved that our path-integral representation can be directly related to that found in [23].

As mentioned above, one interesting and not completely solved problem present in this constrained system is to study the fermionic sector when the double occupancy of lattice sites is excluded. In particular, the role of the fermionic constraints, and thus the fermionic dynamics in the constrained Hilbert space is a crucial problem that must be investigated.

Therefore, one of the main purposes of the present paper is to delve deeper into the discussion of the different alternatives which allows us to define the fermionic propagator in the t–J model.

In [19], within the framework of the perturbative formalism the Feynman rules with appropriate propagators and vertices were found. In particular, a discussion on the fermionic propagator was also given.

In order to continue with the study of fermionic propagation, different alternatives are exposed in the present paper. We show that there is another way of obtaining fermionic propagation to that given in [19]. This is done by working inside the path integral and integrating out the two delta functions on the fermionic constraints.

Moreover, another interesting point is to check our formalism with those obtained by means of the slave-particle representations. More precisely, we will show how our path-integral expression for the partition function (see equation (4.1) of [18]), written in terms of the Hubbard operators, can be mapped in the partition function coming from the slave-fermion representation.

On the other hand, by following the FJ symplectic method, it is possible to show that a new family of first-order constrained Lagrangians written in terms of the Hubbard X-operators exists. This family of classical Lagrangians is able to reproduce the Hubbard X-operator commutation rules, verifying the graded algebra $spl(2, 1)$. As can be shown this family of Lagrangians, totally constrained in the boson-like Hubbard X-operators, can be mapped into the slave-boson representation.

The paper is organized as follow. In section 2, the main results of [18, 19] are collected. In section 3, by analysing the change in the constraints structure of the t–J model, it is shown how starting from our path-integral expression for the partition function found previously, the partition function coming from the decoupled slave-fermion representation can be recovered. In section 4, by using the FJ symplectic method, a different family of first-order constrained Lagrangians is found. This family of Lagrangians corresponds to the situation in which the bosons are totally constrained. In such conditions it is possible to show how the corresponding partition function can be mapped to the partition function coming from the slave-boson representation. In section 5, two alternative ways of defining the fermion propagation are studied. In section 6, conclusions are given.

2. Preliminary and definitions

In the t–J model the three possible states on a lattice site are $|\alpha\rangle = |0\rangle, |+\rangle, |-\rangle$. These states correspond, respectively, to an empty site, an occupied site with a spin-up electron, or an occupied site with a spin-down electron. Double occupancy is forbidden in the t–J model. In terms of these states the Hubbard $\hat{X}$-operators are defined as

$$\hat{X}_i^{\alpha\beta} = |\mathrm{i}\alpha\rangle\langle\mathrm{i}\beta|. \tag{2.1}$$

In equation (2.1), when one of the indices is zero and the other different from zero, the corresponding $\hat{X}$-operator is fermion-like, otherwise boson-like.

The Hubbard $\hat{X}$-operators verify the following graded commutation relations:

$$\left[\hat{X}_i^{\alpha\beta}, \hat{X}_j^{\gamma\delta}\right]_\pm = \delta_{ij}\left(\delta^{\beta\gamma}\hat{X}_i^{\alpha\delta} \pm \delta^{\alpha\delta}\hat{X}_i^{\gamma\beta}\right) \tag{2.2}$$

where the + sign must be used when both operators are fermion-like, otherwise it corresponds to the − sign, and i, j denotes the site indices.

We assume that the family of classical constrained first-order Lagrangians in terms of the Hubbard $\hat{X}$-operators can be written as follows:

$$L = a_{\alpha\beta}(X)\dot{X}^{\alpha\beta} - \boldsymbol{V}^{(0)}. \tag{2.3}$$

In the FJ language [20] the symplectic potential $\boldsymbol{V}^{(0)}$ is defined by

$$\boldsymbol{V}^{(0)} = H(X) + \lambda^a\Omega_a \tag{2.4}$$

where λ^a are appropriate Lagrange multipliers, and so the constraints Ω_a are defined by

$$\Omega_a = \frac{\partial\boldsymbol{V}^{(0)}}{\partial\lambda^a}. \tag{2.5}$$

Therefore, the symplectic supermatrix associated with the Lagrangian (2.3) can be formally written as [22]

$$M_{AB} = \begin{pmatrix} \dfrac{\partial a_{\gamma\delta}}{\partial X^{\alpha\beta}} - (-1)^{|\alpha\beta||\gamma\delta|}\dfrac{\partial a_{\alpha\beta}}{\partial X^{\gamma\delta}} & \dfrac{\partial\Omega_b}{\partial X^{\alpha\beta}} \\ -(-1)^{|a||\gamma\delta|}\dfrac{\partial\Omega_a}{\partial X^{\gamma\delta}} & 0 \end{pmatrix} \tag{2.6}$$

where the compound indices $A = \{(\alpha\beta), a\}$ and $B = \{(\gamma\delta), b\}$ run in the different ranges of the complete set of variables defining the extended configuration space, and $|A|$ indicates the Fermi grading.

Following [18], our starting point is to consider the following partition function for the t–J model written in terms of the four boson-like operators ($X^{+-}, X^{-+}, X^{++}, X^{--}$) and the four fermion-like operators ($X^{0+}, X^{0-}, X^{+0}, X^{-0}$)

$$Z = \int \mathcal{D}X_i^{\alpha\beta}\,\delta(\Omega_{i1})\delta(\Omega_{i2})\delta(\Xi_{i3})\delta(\Xi_{i4})(\mathrm{sdet}\,M_{AB})_i^{1/2}\exp\mathrm{i}\int \mathrm{d}t\,L(X,\dot{X}) \tag{2.7}$$

where $L(X,\dot{X})$ is given by

$$\begin{aligned} L(X,\dot{X}) = \mathrm{i}\sum_i \frac{(1+\rho_i)u_i - 1}{(2-v_i)^2 - 4\rho_i - u_i^2}\left(X_i^{-+}\dot{X}_i^{+-} - X_i^{+-}\dot{X}_i^{-+}\right) \\ +\tfrac{1}{2}\mathrm{i}\sum_{i,\sigma}\left(\dot{X}_i^{0\sigma}X_i^{\sigma 0} + \dot{X}_i^{\sigma 0}X_i^{0\sigma}\right) - \mu\sum_{i,\sigma}X_i^{0\sigma}X_i^{\sigma 0} - H_{t-J}(X) \end{aligned} \tag{2.8}$$

where $u_i = X_i^{++} - X_i^{--}$ and $v_i = X_i^{++} + X_i^{--}$.

The Greek indices α, β takes the values $\{+, -, 0\}$, the index σ takes the values $\{+, -\}$, and $H_{t-J}(X)$ is the usual t–J Hamiltonian

$$H_{t-J} = \sum_{i,j,\sigma} t_{ij} X_i^{\sigma 0} X_j^{0\sigma} + \frac{1}{4} \sum_{i,j,\sigma,\bar{\sigma}} J_{ij} X_i^{\sigma\bar{\sigma}} X_j^{\bar{\sigma}\sigma} - \frac{1}{4} \sum_{i,j,\sigma,\bar{\sigma}} J_{ij} X_i^{\sigma\sigma} X_j^{\bar{\sigma}\bar{\sigma}} \tag{2.9}$$

besides, in equation (2.8) a term depending on the chemical potential μ was added.

In equation (2.7) sdet M_{AB} is the superdeterminant of the symplectic supermatrix M_{AB} defined in (2.6), and the bosonic and fermionic constraints at each site i are given, respectively, by

$$\Omega_{i1} = X_i^{++} + X_i^{--} + \rho_i - 1 = 0 \tag{2.10a}$$

$$\Omega_{i2} = X_i^{+-} X_i^{-+} + \frac{1}{4}(X_i^{++} - X_i^{--})^2 - \left[1 - \frac{1}{2}(X_i^{++} + X_i^{--})\right]^2 + \rho_i = 0 \tag{2.10b}$$

$$\Xi_{i3} = X_i^{0+} X_i^{+-} - X_i^{0-} X_i^{++} = 0 \tag{2.10c}$$

$$\Xi_{i4} = X_i^{+0} X_i^{-+} - X_i^{-0} X_i^{++} = 0 \tag{2.10d}$$

where $\rho_i = X_i^{0+} X_i^{+0} + X_i^{0-} X_i^{-0}$.

In equation (2.8), the Lagrangian coefficients as well as the constraints (2.10) were determined by using the FJ symplectic method with the condition of reproducing the generalized FJ brackets or graded Dirac brackets of the t–J model at the classical level (see [18]).

In particular, the constraint (2.10a) deduced by consistency, is the completeness condition which must be verified by the Hubbard $\hat{X}$-operators and plays an important physical role in this constrained model as will be seen later on. At this stage it is important to remark that in Dirac's language [24] the two bosonic constraints (2.10a) and (2.10b) are second class.

Now, it is useful to write the boson-like Hubbard X-operators in terms of the real components S_α ($\alpha = 1, 2, 3$) of a vector field $\boldsymbol{S}$ and the fermion-like Hubbard X-operators in terms of suitable component spinors (Grassmann variables) [18, 23]

$$X_i^{++} = \frac{1}{2s}(1 - \rho_i)(s + S_{i3}) \tag{2.11a}$$

$$X_i^{--} = \frac{1}{2s}(1 - \rho_i)(s - S_{i3}) \tag{2.11b}$$

$$X_i^{+-} = \frac{1}{2s}(1 - \rho_i)(S_{i1} + \mathrm{i}S_{i2}) \tag{2.11c}$$

$$X_i^{-+} = \frac{1}{2s}(1 - \rho_i)(S_{i1} - \mathrm{i}S_{i2}) \tag{2.11d}$$

$$X_i^{-0} = \Psi_{i+} \qquad X_i^{0-} = \Psi_{i+}^* \tag{2.11e}$$

$$X_i^{+0} = \Psi_{i-} \qquad X_i^{0+} = \Psi_{i-}^* \tag{2.11f}$$

where s is a constant and the hole density in the new variables is written as $\rho_i = \Psi_{i+}^* \Psi_{i+} + \Psi_{i-}^* \Psi_{i-}$. Accounting for the fermionic constraints (2.10) results in $(1 - \rho_i)(1 + \rho_i) = 1$.

The real vector field $\boldsymbol{S}$ can be identified with the spin only when $\rho = 0$, i.e. in the pure bosonic case.

By using the second-class constraints (2.10a) and after the change of variables is made, the partition function takes the form

$$Z = \int \mathcal{D}S_{i1}\,\mathcal{D}S_{i2}\,\mathcal{D}S_{i3}\,\mathcal{D}\Psi_{i\sigma}\,\mathcal{D}\Psi^*_{i\sigma}\,\mathcal{D}\lambda_i\,\mathcal{D}\xi_i\,\mathcal{D}\xi^*_i(\mathrm{sdet}\,M_{AB})_i^{1/2}\left(\frac{\partial X}{\partial S}\right)_i \exp\left(\mathrm{i}\int \mathrm{d}t\,L_{\mathrm{eff}}\right) \tag{2.12}$$

where the quantity $\left(\frac{\partial X}{\partial S}\right)_i$ is the super-Jacobian of the transformation (2.11).

The effective Lagrangian L_{eff} defined in equation (2.12), in terms of the new variables reads

$$\begin{aligned} L_{\mathrm{eff}} = {} & \frac{1}{2s}\sum_i \frac{S_{i1}\dot{S}_{i2} - S_{i2}\dot{S}_{i1}}{s + S_{i3}} + \mathrm{i}\sum_{i,\sigma}\dot{\Psi}^*_{i\sigma}\Psi_{i\sigma} + \mu\sum_{i,\sigma}\Psi_{i\sigma}\Psi^*_{i\sigma} - H_{t-J} \\ & + \sum_i \left[\lambda_i(S^2_{i1} + S^2_{i2} + S^2_{i3} - s^2) + \xi^*_i\left(\Psi_-(S_{i1} - \mathrm{i}S_{i2}) - \Psi_+(s + S_{i3})\right)\right. \\ & \left. + \left(\Psi^*_-(S_{i1} + \mathrm{i}S_{i2}) - \Psi^*_+(s + S_{i3})\right)\xi_i\right] \end{aligned} \tag{2.13}$$

where the Hamiltonian H_{t-J} of the t–J model is written as

$$H_{t-J} = \sum_{i,j,\sigma} t_{ij}\Psi_{i\sigma}\Psi^*_{j\sigma} + \frac{1}{8s^2}\sum_{i,j} J_{ij}(1-\rho_i)(1-\rho_j)\left[S_{i1}S_{j1} + S_{i2}S_{j2} + S_{i3}S_{j3} - s^2\right]. \tag{2.14}$$

In equation (2.13) the parameters λ_i, ξ^*_i and ξ_i are suitable bosonic and fermionic Lagrange multipliers, respectively.

At this stage it is important to remark that our Lagrangian formalism is independent of the underlying lattice dimension.

In the next section we are going to analyse equation (2.7) within the framework of the decoupled slave-particle representations.

3. Slave-particle representations

The standard way of constructing the classical Hamiltonian formulation for slave-particle models and to subsequently give the canonical quantization is developed in [15]. The starting point is to consider the general classical first-order Lagrangian for n bosonic fields b_a and m fermionic fields f_b defined on a lattice

$$L(b^\dagger_a, b_a, f^\dagger_b, f_b) = \tfrac{1}{2}\mathrm{i}\sum_{i,a}(b^\dagger_{ia}\dot{b}_{ia} - \dot{b}^\dagger{}_{ia}b_{ia}) + \tfrac{1}{2}\mathrm{i}\sum_{i,b}(f^\dagger_{ib}\dot{f}_{ib} - \dot{f}^\dagger_{ib}f_{ib}) - H(b^\dagger_a, b_a, f^\dagger_b, f_b). \tag{3.1}$$

Both bosons and fermions fields, are submitted to the slave-particle first-class constraint at each lattice site i,

$$\Omega_i = \sum_a b^\dagger_{ia}b_{ia} + \sum_b f^\dagger_{ib}f_{ib} - 1 = 0. \tag{3.2}$$

Looking at equations (3.1) and (3.2) it can be seen that when the index a takes the values $\pm$ and the index b takes only one value, the six fields (four boson and two fermion fields) define the slave-fermion representation. In contrast, when the index a takes only one value and the index b takes the values $\pm$, the six fields (two boson and four fermion fields) define the slave-boson representation.

With the aim of comparing our results with others obtained previously, in this section we consider the slave-fermion representation. In particular, it is possible to compare the correlation generating functional (2.7) with that obtained from the slave-fermion representation, and this relation is not trivial.

In our approach all the constraints are second class in the Dirac picture [24], while in the slave-particle representations the constraint (3.2) is first class. Thus, when the Hubbard X-operators are decoupled a local gauge symmetry is made evident.

The starting point is the correlation generating functional (2.7) with the Lagrangian (2.8). By computing the $(\mathrm{sdet}\, M_{AB})_i^{1/2}$ appearing in equation (2.7) we find

$$(\mathrm{sdet}\, M_{AB})_i^{1/2} = -\mathrm{i}\frac{(1+\rho_i)}{X_i^{++}} \tag{3.3}$$

where ρ_i evaluated on the constraints is written as

$$\rho_i = \frac{X_i^{0+}X_i^{+0}}{X_i^{++}}.$$

Integrating out the fields components X_i^{--}, X_i^{0-} and X_i^{-0} by using the delta functions on the constraints written as follows:

$$\delta(\Omega_{i2}) = \delta(X_i^{++}X_i^{--} - X_i^{+-}X_i^{-+}) = \frac{1}{X_i^{++}}\delta\left(X_i^{--} - \frac{X_i^{+-}X_i^{-+}}{X_i^{++}}\right) \tag{3.4a}$$

$$\delta(\Xi_{i3}) = \delta(X_i^{0+}X_i^{+-} - X_i^{0-}X_i^{++}) = X_i^{++}\delta\left(X_i^{0-} - \frac{X_i^{0+}X_i^{+-}}{X_i^{++}}\right) \tag{3.4b}$$

$$\delta(\Xi_{i4}) = \delta(X_i^{+0}X_i^{-+} - X_i^{-0}X_i^{++}) = X_i^{++}\delta\left(X_i^{-0} - \frac{X_i^{+0}X_i^{-+}}{X_i^{++}}\right) \tag{3.4c}$$

and taking into account the equality

$$(1+\rho_i)\delta\left(X_i^{-0} - \frac{X_i^{+0}X_i^{-+}}{X_i^{++}}\right) = \delta\left[(1-\rho_i)\left(X_i^{-0} - \frac{X_i^{+0}X_i^{-+}}{X_i^{++}}\right)\right] = \delta\left(X_i^{-0} - \frac{X_i^{+0}X_i^{-+}}{X_i^{++}}\right)$$

(coming from the property of the Grassmann variables), the partition function (2.7) takes the form

$$Z = \int \mathcal{D}X_i^{++}\,\mathcal{D}X_i^{+-}\,\mathcal{D}X_i^{-+}\,\mathcal{D}X_i^{+0}\,\mathcal{D}X_i^{0+}\,\delta\left(X_i^{++} + \frac{X_i^{+-}X_i^{-+}}{X_i^{++}} + \frac{X_i^{0+}X_i^{+0}}{X_i^{++}} - 1\right) \times \exp\left(\mathrm{i}\int \mathrm{d}t\, L^*(X,\dot{X})\right) \tag{3.5}$$

where $L^*(X,\dot{X})$ is given by

$$L^*(X,\dot{X}) = \frac{\mathrm{i}}{2}\sum_i \frac{1}{X_i^{++}}\left(X_i^{-+}\dot{X}_i^{+-} - X_i^{+-}\dot{X}_i^{-+}\right) + \frac{\mathrm{i}}{2}\sum_{i,\sigma}\frac{1}{X_i^{++}}\left(X_i^{+0}\dot{X}_i^{0+} + X_i^{0+}\dot{X}_i^{+0}\right) - H(X). \tag{3.6}$$

As is known the change of variables that allows us to write the remaining five Hubbard X variables in terms of the fields variables in the decoupled slave-fermion representation is defined by

$$X_i^{++} = b_{i+}^{\dagger}b_{i+} \tag{3.7a}$$

$$X_i^{+-} = b_{i+}^{\dagger}b_{i-} \tag{3.7b}$$

$$X_i^{-+} = b_{i-}^{\dagger}b_{i+} \tag{3.7c}$$

$$X_i^{0+} = b_{i+} f_i^{\dagger} \tag{3.7d}$$

$$X_i^{+0} = b_{i+}^{\dagger} f_i. \tag{3.7e}$$

From equations (3.7) it can be seen that the five Hubbard X-fields are given in terms of the six fields of the slave-fermion representation, so it is necessary to introduce an additional condition among the six fields of the slave-fermion representation to make the transformation possible.

We assume the following general linear form for the conditions in each lattice site:

$$\phi_i = \sum_a (G_{ia} b_{ia} + G_{ia}^{\dagger} b_{ia}^{\dagger}) + H_i f_i - H_i^{\dagger} f_i^{\dagger} + K_i = 0 \tag{3.8}$$

where G_{ia}, $G_{ia}^{\dagger}$, K_i are bosonic parameters and H_i, $H_i^{\dagger}$ are fermionic (Grassmannian) parameters.

As was commented above, when the Hubbard X-operators are written in a decoupled representation a local gauge symmetry is made evident. Thus, from a constrained system with a set of second-class constraints, it changes into a constrained system with a first-class constraint, and therefore a gauge-fixing condition must be imposed. Therefore, equation (3.8) is none other than the gauge-fixing condition which corresponds to the local gauge symmetry appearing in the decoupled representation [15].

A convenient choice is to take in equation (3.8): $G_{i+} = i$, $G_{i+}^{\dagger} = -i$ and the remaining coefficients all zero, i.e equation (3.8) reads

$$\phi_i = i(b_{i+} - b_{i+}^{\dagger}) = 0. \tag{3.9}$$

Later on, in equation (3.5) we introduce

$$\begin{aligned} 1 = \int \mathcal{D}b_{i\sigma}^{\dagger}\, \mathcal{D}b_{i\sigma}\, \mathcal{D}f_i^{\dagger}\, \mathcal{D}f_i\, \delta(X_i^{++} - b_{i+}^{\dagger} b_{i+}) \delta(X_i^{+-} - b_{i+}^{\dagger} b_{i-}) \delta(X_i^{-+} - b_{i-}^{\dagger} b_{i+}) \\ \times \delta(X_i^{0+} - f_i^{\dagger} b_{i+}) \delta(X_i^{+0} - b_{i+}^{\dagger} f_i) \delta(\phi_i) J_i \end{aligned} \tag{3.10}$$

where J_i is the super-Jacobian of the transformation (3.7) and (3.9), and its value is $J_i = (b_{i+} + b_{i+}^{\dagger})$.

By integrating out the five variables X_i, the partition function can be written as

$$Z = \int \mathcal{D}b_{i\sigma}^{\dagger}\, \mathcal{D}b_{i\sigma}\, \mathcal{D}f_i\, \mathcal{D}f_i^{\dagger}\, \delta(\Omega_i) \delta(\phi_i) J_i \exp\left(\mathrm{i} \int \mathrm{d}t\, L(b_\sigma, b_\sigma^{\dagger}, f, f^{\dagger}) \right). \tag{3.11}$$

It is easy to see that the super-Jacobian J_i is equal to minus the determinant of the Dirac bracket constructed from the first-class constraint Ω_i and the gauge-fixing condition (3.9), i.e $-\det[\Omega_i, \phi_i]_D$, where the first-class constraint Ω_i in the slave-fermion representation is given by

$$\Omega_i = \sum_{i\sigma} b_{i\sigma}^{\dagger} b_{i\sigma} + f_i^{\dagger} f_i - 1 = 0. \tag{3.12}$$

In equation (3.11) the Lagrangian $L(b_\sigma, b_\sigma^{\dagger}, f, f^{\dagger})$ reads

$$L(b_\sigma, b_\sigma^{\dagger}, f, f^{\dagger}) = \tfrac{1}{2}\mathrm{i} \sum_{i,\sigma} (b_{i\sigma}^{\dagger} \dot{b}_{i\sigma} - \dot{b}^{\dagger}{}_{i\sigma} b_{i\sigma}) + \tfrac{1}{2}\mathrm{i} \sum_i (f_i^{\dagger} \dot{f}_i - \dot{f}_i{}^{\dagger} f_i) - H(b_{i\sigma}^{\dagger}, b_{i\sigma}, f_i^{\dagger}, f_i). \tag{3.13}$$

Therefore, the above considerations show that our correlation-generating functional (2.7) can be mapped into the correlation generating functional (3.11) coming from the slave-fermion representation [15]. This mapping is a consequence of the fermionic constraints present in our expression for the correlation generating functional (2.7). We can also conclude that it is not possible to relate our correlation generating functional (2.7) with that corresponding to the slave-boson representation.

The next question is how to construct from the symplectic FJ formalism a new family of first-order Lagrangian by using the Hubbard X-operators of the graded algebra $spl(2,1)$ as fields variables, in such a way that the results can be mapped in the slave-boson representation. The problem is solved in the next section.

4. Classical Lagrangian and constraints. Slave-boson representation

By following [18, 25], we assume that the family of classical first-order Lagrangians in terms of the Hubbard $\hat{X}$-operators can be written as follows:

$$L = a_{\alpha\beta}(X)\dot{X}^{\alpha\beta} - V^{(0)}. \tag{4.1}$$

where the five Hubbard $\hat{X}$-operators $X^{\sigma\sigma'}$ and X^{00} are boson-like and the four Hubbard $\hat{X}$-operators $X^{\sigma 0}$ and $X^{0\sigma}$ are fermion-like. In the present case the symplectic potential is $V^{(0)} = H(X)$.

The Lagrangian functional coefficients $a_{\alpha\beta}(X)$ that *a priori* are unknown must be determined by consistency in such a way that the graded algebra (2.2) for the Hubbard $\hat{X}$-operators is verified. By following the steps of [18] it is straightforward to construct the symplectic supermatrix associated with the Lagrangian (4.1). Thus the symplectic supermatrix M_{AB} is written in the form

$$M_{AB} = \begin{pmatrix} A_{bb} & B_{bf} \\ C_{fb} & D_{ff} \end{pmatrix}. \tag{4.2}$$

The Bose–Bose parts A_{bb} is a (10×10)-dimensional matrix and it takes the form

$$A_{bb} = \begin{pmatrix} \frac{\partial a_{\sigma\sigma'}}{\partial X^{\sigma''\sigma'''}} - \frac{\partial a_{\sigma''\sigma'''}}{\partial X^{\sigma\sigma'}} & \frac{\partial a_{00}}{\partial X^{\sigma''\sigma'''}} - \frac{\partial a_{\sigma''\sigma'''}}{\partial X^{00}} & \frac{\partial \Omega_{\sigma\sigma'}}{\partial X^{\sigma''\sigma'''}} & \frac{\partial \Omega_{00}}{\partial X^{\sigma''\sigma'''}} \\ -\frac{\partial a_{00}}{\partial X^{\sigma\sigma'}} + \frac{\partial a_{\sigma\sigma'}}{\partial X^{00}} & 0 & \frac{\partial \Omega_{\sigma\sigma'}}{\partial X^{00}} & \frac{\partial \Omega_{00}}{\partial X^{00}} \\ -\frac{\partial \Omega_{\sigma''\sigma'''}}{\partial X^{\sigma\sigma'}} & -\frac{\partial \Omega_{\sigma''\sigma'''}}{\partial X^{00}} & 0 & 0 \\ -\frac{\partial \Omega_{00}}{\partial X^{\sigma\sigma'}} & -\frac{\partial \Omega_{00}}{\partial X^{00}} & 0 & 0 \end{pmatrix}. \tag{4.3}$$

The Bose–Fermi parts B_{bf} (the Fermi–Bose parts $C_{fb} = -B_{bf}^T$) is a (4×10)-dimensional rectangular supermatrix given by

$$B_{bf} = \begin{pmatrix} \frac{\partial a_{0\sigma}}{\partial X^{\sigma''\sigma'''}} - \frac{\partial a_{\sigma''\sigma'''}}{\partial X^{0\sigma}} & \frac{\partial a_{\sigma 0}}{\partial X^{\sigma''\sigma'''}} - \frac{\partial a_{\sigma''\sigma'''}}{\partial X^{\sigma 0}} \\ \frac{\partial a_{0\sigma}}{\partial X^{00}} - \frac{\partial a_{00}}{\partial X^{0\sigma}} & \frac{\partial a_{\sigma 0}}{\partial X^{00}} - \frac{\partial a_{00}}{\partial X^{\sigma 0}} \\ -\frac{\partial \Omega_{\sigma''\sigma'''}}{\partial X^{0\sigma}} & -\frac{\partial \Omega_{\sigma''\sigma'''}}{\partial X^{\sigma 0}} \\ -\frac{\partial \Omega_{00}}{\partial X^{0\sigma}} & -\frac{\partial \Omega_{00}}{\partial X^{\sigma 0}} \end{pmatrix}. \tag{4.4}$$

The Fermi–Fermi parts D_{ff} is the (4×4)-dimensional matrix given by

$$D_{ff} = \begin{pmatrix} \frac{\partial a_{0\sigma}}{\partial X^{0\sigma'}} + \frac{\partial a_{0\sigma'}}{\partial X^{0\sigma}} & \frac{\partial a_{\sigma 0}}{\partial X^{0\sigma'}} + \frac{\partial a_{0\sigma'}}{\partial X^{\sigma 0}} \\ \frac{\partial a_{0\sigma}}{\partial X^{\sigma' 0}} + \frac{\partial a_{\sigma' 0}}{\partial X^{0\sigma}} & \frac{\partial a_{\sigma 0}}{\partial X^{\sigma' 0}} + \frac{\partial a_{\sigma' 0}}{\partial X^{\sigma 0}} \end{pmatrix} \tag{4.5}$$

where $\Omega_{\sigma\sigma'}$ and Ω_{00} are the appropriate bosonic second-class constraints defining the structure of the constrained model.

Once the symplectic algorithm is applied and the correspondent differential equations are solved the solution we found is

$$a_{i0\sigma} = \frac{\mathrm{i}}{2X_i^{00}} X_i^{\sigma 0} \qquad a_{i\sigma 0} = \frac{\mathrm{i}}{2X_i^{00}} X_i^{0\sigma} \tag{4.6}$$

and the boson-like Lagrangian coefficients are all zero.

The set of bosonic second class constraints is given by

$$\Omega_i^{00} = X_i^{00} + X_i^{++} + X_i^{--} - 1 = 0 \tag{4.7a}$$

$$\Omega_i^{\sigma\sigma'} = X_i^{\sigma\sigma'} - \frac{X_i^{\sigma 0} X_i^{0\sigma'}}{X_i^{00}} = 0. \tag{4.7b}$$

In particular, the constraint (4.7*a*) is the completeness condition necessary to avoid double occupancy at each site.

In these conditions the symplectic supermatrix is invertible and the matrix elements of its inverse gives the correct Hubbard graded brackets (2.2), i.e.

$$(M^{AB})^{-1} = -\mathrm{i}(-1)^{|\varepsilon_A|}\big[\hat{A}, \hat{B}\big]_{\pm} \tag{4.8}$$

where $|\varepsilon_A|$ is the Fermi grading of the field variable A.

Consequently, the dynamics in this condition is given by the Lagrangian

$$L(X, \dot{X}) = -\frac{\mathrm{i}}{2} \sum_{i,\sigma} \frac{1}{XX_i^{00}} \big(\dot{X}_i^{0\sigma} X_i^{\sigma 0} + \dot{X}_i^{\sigma 0} X_i^{0\sigma}\big) - H(X). \tag{4.9}$$

The Lagrangian (4.9) together with the bosonic constraints (4.7) correspond to a situation in which the bosons are totally constrained and the dynamics is carried out only by the fermions.

The partition function corresponding to this solution reads

$$Z = \int \mathcal{D}X_i^{\alpha\beta}\, \delta\big[X_i^{00} + X_i^{++} + X_i^{--} - 1\big] \delta\left[X_i^{\sigma\sigma'} - \frac{X_i^{\sigma 0} X_i^{0\sigma'}}{X_i^{00}}\right] (\mathrm{sdet}\, M_{AB})_i^{1/2} \times \exp\left(\mathrm{i} \int \mathrm{d}t\, L(X, \dot{X})\right). \tag{4.10}$$

By computing the superdeterminant of the symplectic matrix appearing in (4.10) we find

$$(\mathrm{sdet}\, M_{AB})_i^{1/2} = \big(\det A \left[\det(D - CA^{-1}B)\right]^{-1}\big)^{1/2} = (X_i^{00})^2. \tag{4.11}$$

Now, in order to confront the correlation generating functional (4.10) with those coming from the slave-boson representation some algebraic manipulations are needed.

The first step is to make the following change of variables:

$$\varphi_{i1} = X_i^{00} - b_i^\dagger b_i = 0 \tag{4.12a}$$

$$\varphi_{i\sigma 0} = X_i^{\sigma 0} - f_{i\sigma}^\dagger b_i = 0 \tag{4.12b}$$

$$\varphi_{i0\sigma} = X_i^{0\sigma} - f_{i\sigma} b_i^\dagger = 0. \tag{4.12c}$$

Analogously to what happens in the slave-fermion representation, in the decoupled slave-boson one, an additional condition among the fields is also needed. From the general linear equation (3.8) we choose by simplicity the following reality condition:

$$\varphi_{i2} = b_i^\dagger - b_i = 0. \tag{4.12d}$$

The super-Jacobian J_i of the transformation (4.12) is given by

$$J_i = -\frac{(b_i^\dagger + b_i)}{(b_i^\dagger b_i)^2}. \tag{4.13}$$

Now, by introducing in equation (4.10) the unity

$$1 = \int \mathcal{D}f_{i\sigma}^\dagger\, \mathcal{D}f_{i\sigma}\, \mathcal{D}b_i^\dagger\, \mathcal{D}b_i\, J_i\, \delta(X_i^{\sigma 0} - f_{i\sigma}^\dagger b_i)\, \delta(X_i^{0\sigma} - f_{i\sigma} b_i^\dagger)\, \delta(b_i^\dagger b_i - X_i^{00})\, \delta(b_i^\dagger - b_i) \tag{4.14}$$

and integrating out all the fields variables $X_i^{\alpha\beta}$, after same algebraic manipulations it is possible to show that the partition function (4.10) takes the form

$$Z = \int \mathcal{D}b_i^\dagger\, \mathcal{D}b_i\, \mathcal{D}f_{i\sigma}^\dagger\, \mathcal{D}f_{i\sigma}\, \delta(\Omega_i)\, \delta(\phi_i)(b_i + b_i^\dagger) \exp\left(\mathrm{i} \int \mathrm{d}t\, L(b^\dagger, b, f_\sigma^\dagger, f_\sigma)\right) \tag{4.15}$$

where Ω_i and ϕ_i are, respectively, the first-class constraint and the gauge-fixing condition in the radial gauge [12], appearing in the partition function of the slave-boson representation [17]. They, respectively, read

$$\Omega_i = b_i^\dagger b_i + \sum_{i,\sigma} f_{i\sigma}^\dagger f_{i\sigma} - 1 = 0. \tag{4.16}$$

$$\phi_i = \mathrm{i}(b_i - b_i^\dagger) = 0. \tag{4.17}$$

Again, we note that the factor $(b + b^\dagger)$ in equation (4.15) is precisely the value of $\det[\Omega_i, \phi_i]_D$ appearing in the gauge theories containing first-class constraints.

The Lagrangian $L(b^\dagger, b, f_\sigma^\dagger, f_\sigma)$ defined in equation (4.15) is given by

$$L(b^\dagger, b, f_\sigma^\dagger, f_\sigma) = \tfrac{1}{2}\mathrm{i} \sum_i (b_i^\dagger \dot{b}_i - \dot{b}^\dagger{}_i b_i) + \tfrac{1}{2}\mathrm{i} \sum_{i,\sigma} (f_{i\sigma}^\dagger \dot{f}_{i\sigma} - \dot{f}_{i\sigma}^\dagger f_{i\sigma}) - H. \tag{4.18}$$

In summary, from our approach and working with the Hubbard X-operators without using any decoupling representation, a new family of Lagrangians (4.9) is obtained. The respective correlation generating functional (4.10) is mapped into the solution provided by the slave-boson representation. It is important to note that the new path integral (4.10) in terms of the Hubbard X-operators, to the best of our knowledge, was developed in the present paper for the first time.

We can see once more how a second-class constrained model written in terms of the Hubbard X-operators, when written in terms of the decoupled slave-particle representations, is transformed into a constrained system where a local gauge symmetry is made evident.

We think that the results we can obtain by using the partition function (4.10) with the Lagrangian (4.9) can be useful for regimes where the system is close to a Fermi liquid state.

In a forthcoming paper the partition function (4.10) will be studied in detail within the context of the perturbative formalism. Having in mind the difficulty of treating the first-class constraint within the path integral (4.15), our purpose is to construct the Feynman rules and the diagrammatics starting from the path integral (4.10). Once an appropriate fermion propagator can be found, our first objective will be to analyse the properties of the fermion spectral function.

5. Two alternative ways to define the propagator of fermion modes

In this section we discuss two alternative ways of treating the fermionic sector in order to define the propagation of the fermion modes. As commented above, a crucial problem in the t–J model is to define the fermionic propagation in the constrained Hilbert space, when double occupancy is forbidden.

With the purpose of studying this problem, in [19] the correlation generating functional (2.12) was considered at finite temperature by means of the 'Euclideanization procedure'. Moreover, it was assumed that we are close to an undoped regime where the system is an antiferromagnetic insulator. Under this condition there are a small number of holes and it can be assumed that the hole density $\rho_i = \langle \rho_i \rangle =$ constant. The constant value ρ of the hole density must be determined later on by consistency, for a given value of the chemical potential μ.

In these conditions, it is possible to treat the non-polynomic Lagrangian (2.13) within the framework of the perturbative formalism, and so it can be partitioned as follows:

$$L_{\rm eff} = L^B(\boldsymbol{S}, \lambda) + L^F(\boldsymbol{\eta}) + L^I(\boldsymbol{S}, \boldsymbol{\eta}) \tag{5.1}$$

where

$$\begin{aligned} L^B(\boldsymbol{S}, \lambda) = &-\frac{\mathrm{i}}{2s}\sum_i \frac{\widetilde{S}_{i1}\dot{\widetilde{S}}_{i2} - \widetilde{S}_{i2}\dot{\widetilde{S}}_{i1}}{s+s'} + 2s'\sum_i \lambda_i \widetilde{S}_{i3} \\ &+\frac{1}{8s^2}\sum_{i,I} J'\left[\widetilde{S}_{i1}\widetilde{S}_{(i+I)1} - \widetilde{S}_{i2}\widetilde{S}_{(i+I)2} - \widetilde{S}_{i3}\widetilde{S}_{(i+I)3} + \widetilde{S}_i^2\right] \end{aligned} \tag{5.2a}$$

and

$$L^F(\boldsymbol{\eta}) + L^I(\boldsymbol{S}, \boldsymbol{\eta}) = \sum_{i,\sigma} \dot{\Psi}^*_{i\sigma}\Psi_{i\sigma} + \mu\sum_{i,\sigma}\Psi_{i\sigma}\Psi^*_{i\sigma} + \sum_{i,j,\sigma} t_{ij}\Psi_{i\sigma}\Psi^*_{j\bar{\sigma}} + \sum_i \bar{\boldsymbol{\eta}}_i \mathcal{M}_i \boldsymbol{\eta}_i. \tag{5.2b}$$

Considering the bilinear bosonic part of equation (2.13), for a constant value of the hole density and taking $J_{ij} =$ constant, we arrive at equation (5.2a). In equation (5.2a) was defined $J' = J(1-\rho)^2$. Moreover, the symbol $\sum_I$ indicates a sum over nearest-neighbour sites.

Besides, in equation (5.2a) it was assumed that the vector $\boldsymbol{S}$ is written as

$$\boldsymbol{S} = (0, 0, s') + (\widetilde{S}_1, \widetilde{S}_2, \widetilde{S}_3) \tag{5.3}$$

where $\widetilde{S}_1, \widetilde{S}_2, \widetilde{S}_3$ are the fluctuations. So, equation (5.2a) corresponds to the lowest order of the system fluctuating around an antiferromagnetic state. Moreover, we must consider $s' \neq s$ because as is known the local magnetization in an antiferromagnetic state is reduced from its classical value, even for the pure Heisenberg model. The value of s' must also be determined by consistency.

In equation (5.2b) the four-component spinor $\boldsymbol{\eta} = \binom{\boldsymbol{\Psi}}{\boldsymbol{\xi}}$, is constructed from the two spinors $\boldsymbol{\Psi}$ and $\boldsymbol{\xi}$. The physical two-component spinor $\boldsymbol{\Psi} = \binom{\Psi_+}{\Psi_-}$ is restricted by the fermionic constraint equations (2.10c), (2.10d), and the two-component spinor $\boldsymbol{\xi} = \binom{\xi_+}{\xi_-}$ is a Majorana spinor.

In the same equation the (4×4)-dimensional matrix $\mathcal{M}$ is defined by

$$\mathcal{M} = \begin{pmatrix} 0 & \boldsymbol{I}s + \boldsymbol{S}\cdot\boldsymbol{\sigma} \\ \boldsymbol{I}s + \boldsymbol{S}\cdot\boldsymbol{\sigma} & 0 \end{pmatrix} \tag{5.4}$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices.

In this regime the diagrammatics and the Feynman rules can be found. In particular, the bilinear part of the bosonic sector written in equation (5.2*a*) gives rise to the usual antiferromagnetic magnon propagator (see [19]).

The bilinear fermionic part $L^F(\boldsymbol{\eta})$ of equation (5.2*b*) can be written in terms of the four-component spinor $\boldsymbol{\eta}$ and it is given by

$$L^F(\boldsymbol{\eta}) = \sum_{i,j} \bar{\eta}_{i\alpha} (G^{-1}_{(0)ij})^{\alpha\beta} \eta_{j\beta} \tag{5.5}$$

where in the Fourier space the symmetric non-singular (4×4)-dimensional matrix $G^{-1}_{(0)}$ is defined by

$$(G^{\alpha\beta}_{(0)})^{-1}(k, \nu_n, \nu'_n) = \begin{pmatrix} -(\mathrm{i}\nu_n + \mu) & \varepsilon_k & \frac{1}{2}(s+s') & 0 \\ \varepsilon_k & -(\mathrm{i}\nu_n + \mu) & 0 & \frac{1}{2}(s-s') \\ \frac{1}{2}(s+s') & 0 & f & g \\ 0 & \frac{1}{2}(s-s') & g & -f \end{pmatrix} \delta(\nu_n, \nu'_n). \tag{5.6}$$

In equation (5.6) the quantities k and ν_n are, respectively, the momentum and the Matsubara frequency of the fermionic field, and was defined as $\varepsilon_k = -t \sum_{I} \exp(-\mathrm{i}\boldsymbol{I} \cdot \boldsymbol{k})$.

The functions f and g appearing in equation (5.6) are totally arbitrary. As can be easily seen, these functions do not appear in the Lagrangian due to the Majorana condition on the two-component spinor $\boldsymbol{\xi}$.

In this scenario, the symmetric matrix defining the fermionic free propagator $G_{(0)\alpha\beta}$ is given by the inverse of the matrix (5.6). The physical components of the free propagator are given by the matrix elements $G_{(0)11}$, $G_{(0)12}$, $G_{(0)21}$ and $G_{(0)22}$ and they were explicitly written and analysed in section 4 of [19]. So, the Feynman rules propagators and vertices are given straightforwardly and therefore the boson and fermion self-energy can be computed.

However, some particular features of the fermionic free propagator must be emphasized.

The trick of introducing an auxiliary two-component Majorana spinor is a way to obtain a free functional $G_{(0)}$ that really propagates physical fermionic modes.

The electron spectral function is defined from the fermionic propagator $G_{(0)\alpha\beta}$ by considering the components $G_{(0)11}$ and $G_{(0)22}$. The matrix elements directly connected with the electronic properties, such as, for example, the Fermi surface (FS), are precisely $G_{(0)11}$ and $G_{(0)22}$. The electronic spectral function measured in photoemission experiments [26] must be related to minus the imaginary part of these matrix elements.

In contrast to the fermion propagator obtained by means of the standard Green function method [27], our fermion propagator contains two poles. It is important to say that for a given filled factor the chemical potential we obtain is exactly the same as that obtained by using the standard Green function method.

By plotting the electron spectral function (see [19, figure 1]) we can see that the peak at negative energy must be interpreted as the extraction of an electron, while the peak at positive energy represents the addition of an electron to the system. Therefore, the two peaks account for the photoemission and the inverse of the photoemission, respectively. The presence of these two peaks implies that for a given value of k, the state is not completely filled or empty. Photoemission experiments are only sensitive to the first peak. Then the first peak of our propagator must be related to the excitation measures in photoemission.

An alternative way to treat the fermionic sector is to start from equation (2.7) and to work inside the path integral. So, by integrating out the two delta functions on the fermionic constraints Ξ_{i3} and Ξ_{i4}, the partition function can be written as follows:

$$Z = \int \mathcal{D}X_i^{\sigma\sigma'} \mathcal{D}X_i^{0+} \mathcal{D}X_i^{+0}\, \delta(\Omega_{i1})\, \delta(\Omega_{i2}) (\mathrm{sdet}\, M_{AB})_i^{1/2} \exp \mathrm{i} \int \mathrm{d}t\, L^*(X, \dot{X}) \tag{5.7}$$

where $L^*(X, \dot{X})$ is given by

$$L^*(X,\dot{X}) = \frac{\mathrm{i}}{2}\sum_i \frac{1}{X_i^{++}}\left(X_i^{-+}\dot{X}_i^{+-} - X_i^{+-}\dot{X}_i^{-+}\right) + \frac{\mathrm{i}}{2}\sum_{i,\sigma}\frac{1}{X_i^{++}}\left(\dot{X}_i^{0+}X_i^{+0} + \dot{X}_i^{+0}X_i^{0+}\right) - H(X). \tag{5.8}$$

The total Hamiltonian H is defined by

$$H = H_{t-J} + \mu\sum_{i,\sigma} X_i^{0\sigma}X_i^{\sigma 0} \tag{5.9}$$

where the Hamiltonian H_{t-J} defined in equation (2.9) must be evaluated on the fermionic constraints Ξ_{i3} and Ξ_{i4}.

Due to the nonlinearity of the constraints (2.10*c*) and (2.10*d*), when the path integration on the two fermionic fields X_i^{0-} and X_i^{-0} is carried out, the non-polynomial structure of the kinetic fermionic part of the Lagrangian is made evident, as can be seen from equation (5.8).

After the four boson-like X-Hubbard operators are related to the real components S_α ($\alpha = 1, 2, 3$) of a vector field $\boldsymbol{S}$ and the remaining two fermion-like X-Hubbard operators are written in terms of suitable component spinor fields (see equations (2.11)), the correlation generating functional (5.7) takes the form

$$Z = \int \mathcal{D}S_{i1}\,\mathcal{D}S_{i2}\,\mathcal{D}S_{i3}\,\mathcal{D}\Psi_{i-}\,\mathcal{D}\Psi_{i-}^*\,\mathcal{D}\lambda_i\,(\mathrm{sdet}\,M_{AB})_i^{1/2}\left(\frac{\partial X}{\partial S}\right)_i \exp\left(\mathrm{i}\int \mathrm{d}t\,L_{\mathrm{eff}}\right). \tag{5.10}$$

Now, the Lagrangian L_{eff} defined in equation (5.10) is given by

$$L_{\mathrm{eff}} = \frac{1}{2s}\sum_i (1-\rho_i)\left(\frac{S_{i1}\dot{S}_{i2} - S_{i2}\dot{S}_{i1}}{s+S_{i3}}\right) + \mathrm{i}s\sum_i \frac{1}{s+S_{i3}}\left(\dot{\Psi}_{i-}^*\Psi_{i-} + \dot{\Psi}_{i-}\Psi_{i-}^*\right)$$
$$-H + \sum_i \lambda_i\left(S_{i1}^2 + S_{i2}^2 + S_{i3}^2 - s^2\right) \tag{5.11}$$

where in equation (5.11) and hereafter the tilde over the fluctuations is omitted.

The Hamiltonian H written in terms of the real vector variable $\boldsymbol{S}$ and the two-spinor-component fields Ψ_{-i} and Ψ_{-i}^*, reads

$$H = \sum_{i,j}\frac{t_{ij}}{(s+S_{i3})(s+S_{j3})}\left[S_{i1}S_{j1} + S_{i2}S_{j2} + S_{i3}S_{j3} + s^2 + s(S_{i3}+S_{j3})\right.$$
$$\left. + \mathrm{i}(S_{i1}S_{j2} - S_{i2}S_{j1})\right]\Psi_{-i}\Psi_{-j}^* + \frac{1}{8s^2}\sum_{i,j}J_{ij}(1-\rho_i)(1-\rho_j)$$
$$\times\left[S_{i1}S_{j1} + S_{i2}S_{j2} + S_{i3}S_{j3} - s^2\right] + 2s\mu\sum_i\left(\frac{1}{s+S_{i3}}\right)\Psi_{-i}\Psi_{-i}^*. \tag{5.12}$$

Again, the path integral (5.8) is considered within the framework of the perturbative formalism at finite temperature, and we assume that we are close to an undoped regime (an antiferromagnetic insulator).

After a rotation of spins on the second sublattice by 180° about the S_1-axis is performed, the Euclidean and rotated Lagrangian L_{eff}^{ER} is obtained, and so the lowest order of the effective Lagrangian (5.11) can be partitioned as follows

$$L_{\mathrm{eff}}^{ER} = L^B(\boldsymbol{S},\lambda) + L^F(\Psi_{-i},\Psi_{-i}^*) + L^I(\boldsymbol{S},\Psi_{-i},\Psi_{-i}^*) \tag{5.13}$$

where

$$L^B(\boldsymbol{S},\lambda) = -\frac{\mathrm{i}}{2s}(1-\rho)\sum_i \frac{S_{i1}\dot{S}_{i2} - S_{i2}\dot{S}_{i1}}{s+s'} + 2s'\sum_i \lambda_i S_{i3}$$
$$+\frac{1}{8s^2}\sum_{i,I} J'\left[S_{i1}S_{(i+I)1} - S_{i2}S_{(i+I)2} - S_{i3}S_{(i+I)3} + S_i^2\right] \qquad (5.14a)$$

and

$$L^F = \frac{s}{s+s'}\sum_i(\dot{\Psi}^*_{i-}\Psi_{i-} + \dot{\Psi}_{i-}\Psi^*_{-i}) + \frac{2\mu s}{s+s'}\sum_i \Psi_{i-}\Psi^*_{i-} \qquad (5.14b)$$

$$L^I = \sum_{i,j}\frac{t_{ij}}{s+s'}\left(S_{i1} - \mathrm{i}S_{i2} + S_{j1} + \mathrm{i}S_{j2}\right)\Psi_{i-}\Psi^*_{j-}. \qquad (5.14c)$$

At this stage it is important to note that our effective theory does not contain fermion dispersion. This feature is also present in the slave-fermion theories when the fermion dynamics is generated *via* interaction with virtual magnons.

By making a Fourier transformation it is possible to see that the bilinear bosonic part of the Lagrangian (5.14*a*) allows us to recover the structure of the bosonic propagator (antiferromagnetic magnons) given by

$$\mathcal{D}^{ab}_{(0)}(q,\omega_n,\omega'_n) = \begin{pmatrix} \frac{J'z}{4s^2 d_{(0)}}(1-\gamma_q) & -\frac{2\omega_n}{(s+s')d_{(0)}}(1-\rho) & 0 & 0 \\ \frac{2\omega_n}{(s+s')d_{(0)}}(1-\rho) & \frac{J'z}{4s^2 d_{(0)}}(1+\gamma_q) & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2s'} \\ 0 & 0 & \frac{1}{2s'} & -\frac{J'z(1-\gamma_q)}{16s^2 s'^2} \end{pmatrix}$$
$$\times\delta(\omega_n,\omega'_n). \qquad (5.15)$$

In equation (5.15) the quantity $d_{(0)}$ is given by

$$d_{(0)} = \left(\frac{2(1-\rho)}{s+s'}\right)^2\left(\omega_q^2 + \omega_n^2\right) \qquad (5.16)$$

where the frequency ω_q^2 is defined by

$${\omega_q}^2 = \left[\frac{zJ'}{4s^2}\left(\frac{s+s'}{2(1-\rho)}\right)\right]^2(1-\gamma_q^2). \qquad (5.17)$$

Moreover, in equation (5.15) z is the number of nearest neighbours and was defined the quantity $\gamma_q = \frac{1}{z}\sum_I \exp(\mathrm{i}\boldsymbol{I}\cdot\boldsymbol{q})$.

Analogously, the bilinear fermionic part (5.14*b*) reads

$$L^F = \sum_{k,\nu_n}\Psi^*_-(k,\nu_n)G_0^{-1}\Psi_-(k,\nu_n) \qquad (5.18)$$

where we have defined

$$G_0^{-1} = \frac{2s}{s+s'}(\mathrm{i}\nu_n - \mu). \qquad (5.19)$$

The inverse of this scalar function given by

$$G_0 = \frac{s+s'}{2s}\frac{1}{\mathrm{i}\nu_n - \mu} \tag{5.20}$$

is a (non-propagating) functional which only depends on the Matsubara frequency ν_n.

Finally, in the approximation that we consider the unique three-leg vertex is defined by

$$U_a = \frac{1}{s+s'}\begin{pmatrix} \varepsilon(k')+\varepsilon(k) \\ \mathrm{i}(\varepsilon(k')-\varepsilon(k)) \\ -\dfrac{s}{s+s'}[\mathrm{i}(\nu+\nu')-2\mu] \\ 0 \end{pmatrix}\delta(q+k-k')\delta(\omega+\nu-\nu'). \tag{5.21}$$

At this point, the problem is to analyse the bilinear fermionic sector, in order to give the prescriptions for the propagation of the fermionic modes. The usual way to solve the propagation of fermions is by means of the Dyson equation. As known the Dyson theorem allows us to compute the inverse of the corrected fermion propagator in terms of the free-fermion propagator and the self-energy. Therefore, the propagator $G(k,\nu_n) = [G_0^{-1}(\nu_n) - \Sigma(k,\nu_n)]^{-1}$ can be calculated in a straightforward way within the self-consistent Born approximation [28, 29]. By using standard techniques the following expression for the self-energy at zero temperature is found

$$\Sigma(k,\mathrm{i}\nu_n) = \frac{(1+\rho)}{2N}t^2z^2\sum_q \frac{\left[\gamma_k[1-(1-\gamma_q^2)^{1/2}]^{1/2} - \gamma_{k+q}[1+(1-\gamma_q^2)^{1/2}]^{1/2}\right]^2}{(1-\gamma_q^2)^{1/2}} \times\frac{1}{\mathrm{i}\nu_n - \omega_q - \mu - \Sigma(k+q,\mathrm{i}\nu_n-\omega_q)}. \tag{5.22}$$

The expression (5.22) is useful in the strong-coupling case ($t > J$). On the other hand, the self-consistent solution of this equation is necessary in order to obtain fermionic propagation, and it must be performed numerically. Once an appropriate self-energy function $\Sigma(k,\mathrm{i}\nu_n)$ is found the propagator $G(k,\nu)$ remains well defined and it is possible to compute numerically the spectral function defined by $A(k,\nu) = -\frac{1}{\pi}\lim_{\varepsilon\to 0} G(k,\nu+\mathrm{i}\varepsilon)$. Finally, as well known, the correction to the bosonic propagator is given by

$$\mathcal{D}_{ab} = \left[\mathcal{D}_{(0)ab}^{-1} - \Pi_{ab}\right]^{-1} \tag{5.23}$$

where $\mathcal{D}_{(0)ab}^{-1}$ is the inverse of the free bosonic propagator (5.15) and the bosonic self-energy Π_{ab} reads

$$\Pi_{ab}(q,\omega_n) = \left(\frac{s+s'}{2s}\right)^2\sum_{k,\nu_n}\frac{U_aU_b}{[\mathrm{i}\nu_n+\mu-\Sigma(k,\nu_n)][\mathrm{i}(\nu_n+\omega_n)+\mu-\Sigma(q+k,\nu_n+\omega_n)]}. \tag{5.24}$$

As can be seen from the above equation the dimension of the underlying lattice and the physics depend on the parameters z, γ_q and ε_k, though our general formalism is dimension independent.

It is important to confront our results with others given previously in the literature related to the spin-polaron theories [28]. Like in these theories our starting point was to assume an antiferromagnetic order state. This physical assumption is directly connected with the fact that at lowest order the fermion is not propagating (see equation (5.20)). Then, in order to describe a metallic phase where the holes move coherently on the lattice, it is necessary to solve the self-consistent equation (5.22).

The solution of our equations (5.22) and (5.24) together with a quantitative comparison with the spin-polaron theories is an important matter that deserves further study.

Another point to take into account in a future work is to study the relationship between our matrix propagator $G_{(0)\alpha\beta}$ (see equation (4.13) of [19]) and those obtained by solving equation (5.22) self-consistently.

As is known, any model or approach will be considered as a good candidate to describe high-T_c superconductors when it is able to answer the question of why the antiferromagnetic long-range order disappears for small values of doping (for instance, $\rho = 0.04$–0.05). In the last few years this problem was attacked from different approaches [13, 14, 30]. In a future work and from our formalism, we will also have a response to give concerning this important point related to the disappearance of the antiferromagnetism.

In the present section, the magnetic excitations that we have considered are antiferromagnetic magnons and in addition we have assumed a strong long-range antiferromagnetic order. Therefore, our next step must be to study the instability of the antiferromagnetic order and to analyse against which phase this is unstable. In order to have some idea about this fact, in [19] we have studied the magnon self-energy effects on the magnetic spectral function. Besides the softening of the antiferromagnetic magnon we have also found a reduction of the magnetic spectral signal. These results were obtained using our two pole bare fermionic propagator. In the near future and in order to improve our calculation we will solve a self-energy coupled problem for both magnetic and electronic dynamics.

6. Conclusions

As shown first in [25] for the pure bosonic case ($su(2)$ algebra), in a classical Lagrangian formalism it is not possible to introduce the full Hubbard algebra by means of constraints. Consequently, in a path-integral formulation complete information about the Hubbard algebra cannot be introduced; namely, the commutation rules, the completeness condition and the multiplication rules for the Hubbard X-operators. So, the Heisenberg model treated in the Lagrangian picture only admits two second-class constraints, and these are the completeness condition $X^{++} + X^{--} + X^{00} = 1$ and the nonlinear constraint $X^{+-} X^{-+} + \frac{1}{4}(X^{++} - X^{--})^2 = s^2$.

The latter constraint in not really the group Casimir operator. It can be shown that the presence of such a constraint is consistent with the quantization of a spin system in the limit of large spin s, or equivalently for magnetic order state.

A similar situation actually occurs in the case in which the Hubbard X-operators verify the graded algebra $spl(2, 1)$, but in this case at least two solutions are possible. When the Hubbard X-operators close the graded algebra $spl(2, 1)$ the t–J model described in terms of a first-order Lagrangian has the following possible solutions.

(a) One is the family of first-order Lagrangians (2.8) together with the set of second-class constraints (2.10). Two of them are bosonics and the other two are fermionics. In particular, the constraint (2.10*a*) is the completeness condition. As was shown the corresponding path-integral formalism is mapped into the decoupled slave-fermion representation. So, in this case our correlation generating functional (2.7) favours the magnon dynamics of the system with a strong magnetic order state feature (consistent with the large-s nature of the constraint (2.10*b*)).

(b) The different family of first-order Lagrangians (4.9) together with the new set of second-class constraints (4.7) is also a possible solution. In this case all the constraints are bosonic, and (4.7*a*) is again the completeness condition. As it can be seen the remaining four constraints (4.7*b*) are related to the multiplication rules. In this situation the bosons are totally constrained and the dynamics is carried out only by the fermions. Moreover, we note that no nonlinear constraint of the type (2.10*b*) appears. As was shown the path-integral formalism corresponding to this dynamical situation is mapped in the slave-boson representation. Therefore, in our correlation-generating functional (4.10) the fermion dynamics with a strong feature of Fermi liquid is preferred.

It is possible to conclude that once the set of second-class constraints is chosen, different families of Lagrangians are obtained, and so we can ensure that each family contains different physics.

It is worthwhile to remark that both Lagrangian formalism are independent of the dimension of the underlying lattice.

Moreover, as was seen in all the cases the completeness condition appears as necessary. As is well known the completeness condition involves an important physical meaning. Such a condition avoids the double occupancy at each lattice site at the quantum level.

Finally, in section 5 two alternative ways of defining the fermion propagator were developed. By means of the trick of introducing auxiliary Majorana spinors, a free fermion matrix propagator having two poles was found. Later on, by integrating out two of the fermions using the delta functions, it was possible to obtain the non-propagating scalar function (5.20) in the fermionic sector. From this free 'propagator' and by means of the Dyson equation the fermion self-energy can be evaluated straightforwardly within the self-consistent Born approximation.

Since our path-integral formalism is mapped in the slave-fermion formalism and taking into account that equation (5.22) for the self-energy at zero temperature has a similar structure to that obtained from the spin-polaron theories [28], both results allow us to ensure that our approach is consistent.

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